Nongray Error in Total Emittance Measurements

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In total emittance measurements, the analysis generally assumes that the sample absorptance to the surrounds radiation is equal to the sample emittance. The accuracy of this assumption depends upon the sample spectral reflectance in the wavelength region corresponding to the emitted energy spectrum and to the surrounds radiation energy spectrum. The nongray error is a result of the assumption of absorptance equals emittance. Edwards and Nelson[1] have discussed this error and have presented values of the maximum discrepancy in the experimental total emittance. The title of their paper implies the maximum percentage error occurs for the same conditions as the maximum discrepancy. This is not the case, however.

In the following discussion, the nongray error will be derived in general terms and numerically evaluated for certain idealized but limiting, spectral reflectance variations with wavelength. The results provide bounds upon the error contributed to the experimental measurements from the nongrayness of materials. The expression for the determination of the total emittance of a material by either the calorimetric or radiometric method will be of the form:

$$\epsilon_s A_s \sigma T_s^4 - \alpha_s A_s \sigma T_0^4 = Q \tag{1}$$

where

ϵ_s = sample emittance at temperature T_s
A_s = sample area
T_s = sample temperature
α_s = sample absorptance at temperature T_s to surrounds radiation at an effective temperature T_0
T_0 = effective surrounds radiant temperature
Q = heat lost by the sample as measured radiometrically or calorimetrically
σ = Stefan-Boltzman constant

In order to determine the importance of the assumption that absorptance equals emittance, the difference between absorptance and emittance can be written as

$$\Delta\alpha_s = \alpha_s - \epsilon_s \tag{2}$$

The contribution to the total error from the terms containing $\Delta\alpha_s$ can be obtained from Eqs. (1) and (2). Assuming the errors to be linear and additive, an error analysis yields the following expression for the nongray portion:

$$\left[\frac{\delta\epsilon_s}{\epsilon_s}\right]_{NG} = \left(\frac{\Delta\alpha_s}{\epsilon_s}\right)\left(\frac{T_0^4}{T_s^4 - T_0}\right)\left\{4\,\frac{\delta T_0}{T_0} + \frac{\delta A_s}{A_s} + \frac{\delta\Delta\alpha_s}{\Delta\alpha_s}\right\} \tag{3}$$

where

δ = error in the measurement or evaluation of a quantity
$[\delta\epsilon_s/\epsilon_s]_{NG}$ = fractional error in emittance contributed by the assumption of $\alpha_s = \epsilon_s$, i.e., the nongray error

The first two quantities in the braces of Eq. (3) are experimentally determinable or controllable errors and, hence, should be of the order of a few percent or less. The third term in these braces can be considered to be of the order of unity when the assumption that absorptance equals emittance is made. Consequently, the magnitude of the first quantity in the braces is dominant and can be considered to be controlling. The nongray error given by Eq. (3), therefore, can be written as

$$\left[\frac{\delta\epsilon_s}{\epsilon_s}\right]_{NG} \geq \left(\frac{\Delta\alpha_s}{\epsilon_s}\right)\left(\frac{T_0^4}{T_s^4 - T_0^4}\right) \tag{4}$$

The emittance and absorptance of a surface is defined as

$$\epsilon_s = \left[\int_0^\infty \epsilon_\lambda\, E(\lambda,T_s)d\lambda\right] \Big/ \sigma T_s^4 \tag{5}$$

$$\alpha_s = \left[\int_0^\infty \alpha_\lambda\, E(\lambda,T_0)d\lambda\right] \Big/ \sigma T_0^4 \tag{6}$$

where

$$E(\lambda,T) = \frac{C_1\lambda^{-5}}{\exp(C_2/\lambda T) - 1} \text{ (Planck's equation)} \tag{7}$$

ϵ_λ = monochromatic emittance
α_λ = monochromatic absorptance
C_1,C_2 = first and second radiation constant
λ = wavelength

The resulting expression for the nongray error obtained from Eqs. (2, and 4–6) is

$$\frac{\delta\epsilon_s}{\epsilon_{s_{NG}}} \geq \frac{T_0^4}{T^4 - T_0^4} \times \left[\frac{\left\{\left[\int_0^\infty \alpha_\lambda E(\lambda,T_0)d\lambda\right]\Big/T_0^4\right\} - \left\{\left[\int_0^\infty \epsilon_\lambda E(\lambda,T_s)d\lambda\right]\Big/T_s^4\right\}}{\left[\int_0^\infty \epsilon_\lambda E(\lambda,T_s)d\lambda\right]\Big/T_s^4}\right] \tag{8}$$

The evaluation of Eq. (8) requires a knowledge of the variation of spectral absorptance or emittance with wavelength. Three hypothetical spectral characteristics can be used to bound the nongray error:

Case 1	$\alpha_\lambda = \epsilon_\lambda = 1$	$\lambda < \lambda_c$	(9a)
	$\alpha_\lambda = \epsilon_\lambda = 0$	$\lambda > \lambda_c$	
Case 2	$\alpha_\lambda = \epsilon_\lambda = 0$	$\lambda < \lambda_c$	(9b)
	$\alpha_\lambda = \epsilon_\lambda = 1$	$\lambda > \lambda_c$	
Case 3	$\alpha_\lambda = \epsilon_\lambda = 0$	$\lambda < \lambda_c$	
	$\alpha_\lambda = \epsilon_\lambda = 1$	$\lambda = \lambda_c$	(9c)
	$\alpha_\lambda = \epsilon_\lambda = 0$	$\lambda > \lambda_c$	

Cases 1 and 2 are "step" spectral emittance characteristics, and case 3 is a spectral characteristic corresponding to a

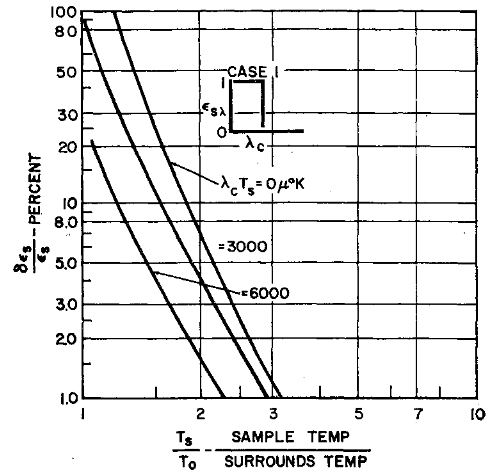

Fig. 1 Nongray error for selective absorber-type material.

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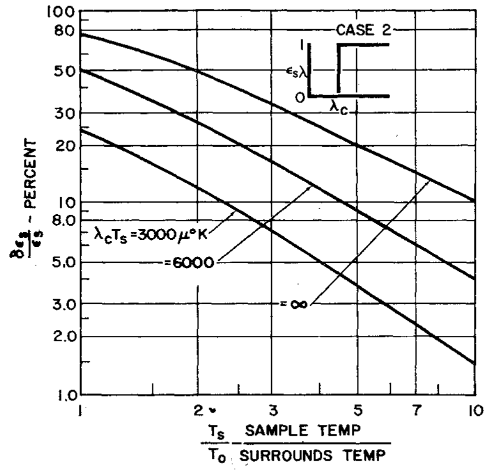

Fig. 2 Nongray error for selective emitter-type material.

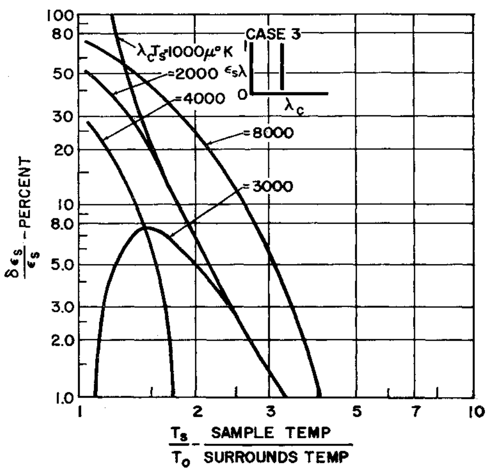

Fig. 3 Nongray error for spike emitter-type material.

"spike" or single emittance line. Edwards and Nelson[1] presented partial results for case 1.

Making the substitutions

$$F(Z) = C_1Z^{-5}/\{[\exp(C_2/Z)] - 1\} = T^{-5}E(\lambda,T) \quad (10a)$$

$$Z = \lambda T \quad (10b)$$

$$T^* = T_s/T_0 \quad (10c)$$

in Eq. (8) and applying the limits from Eq. (9), the nongray error for the cases considered becomes

Case 1

$$\frac{\delta\epsilon_s}{\epsilon_s} \geq \frac{1}{[(T^*)^4 - 1]}\left[\int_{\lambda_cT_0}^{\lambda_cT_0T^*} F(Z)dZ \Big/ \int_0^{\lambda_cT_0T^*} F(Z)dZ\right] \quad (11a)$$

Case 2

$$\frac{\delta\epsilon_s}{\epsilon_s} \geq \frac{1}{[(T^*)^4 - 1]}\left[\int_{\lambda_cT_0}^{\lambda_cT_0T^*} F(Z)dZ \Big/ \int_{\lambda_cT_0T^*}^{\infty} F(Z)dZ\right] \quad (11b)$$

Case 3

$$\frac{\delta\epsilon_s}{\epsilon_s} \geq \frac{1}{[(T^*)^4 - 1]}\;\frac{F(\lambda_cT_0) - T^*F(\lambda_c,T_0,T^*)}{T^*F(\lambda_c,T_0,T^*)} \quad (11c)$$

The results of numerical computation of Eq. (11) are shown in Figs. 1–3 for a range of T_s^* and λ_cT_s. If λ_cT_s is equal to 3000, the material has an emittance of 0.27 if it is a case 1 material and 0.73 if it is a case 2 material; if the value of λ_cT_s is 6000, the corresponding emittances are 0.73 and 0.27, respectively. The extremes of λ_cT_s cannot be ignored, however. For both case 1 and 2 materials, the extremes of λ_cT_s are highly reflective materials, and these extremes indicate the possible effect of a surface film that is transparent through almost all of the wavelengths involved in the emission. The nongray error can be minimized by maintaining a large value of T_s^* in the instance of case 1 or 3. The error for case 2 is significant for values of T_s^* as large as 5–10.

Reference

[1] Edwards, D. K. and Nelson, K. E., "Maximum error in total emissivity measurements due to nongrayness of samples," ARS J. **31**, 1021–1022 (1961).

Erratum: "Electrical Discharge Across a Supersonic Jet of Plasma in Transverse Magnetic Field"

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Line 7 of the above paper should read "... at a velocity of approximately 3×10^3," rather than "... 3×10^4." The value was correct in galley proof, but the error occurred in making another correction in this paragraph in page proof.

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Moment of Momentum by Direction Cosines

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The equation relating the sum of the moments acting on a rigid body to the time rate of change of its moment of momentum can be written

$$\mathbf{M} = [C_{01}][\dot{C}_{10}][I]_1([C_{01}][\dot{C}_{10}]) + [I]_1([\dot{C}_{01}][\dot{C}_{10}] + [C_{01}][\ddot{C}_{10}]) + m\,\mathbf{r}_1 \times \mathbf{a} \quad (1)$$

where $X_0Y_0Z_0$ is an inertial Cartesian coordinate system and $X_1Y_1Z_1$ is *any* coordinate system fixed in the body. Its origin is 0_1.

$[C_{01}]$ is the direction cosine matrix taken from the transformation matrix that transforms the coordinates of a point *from* system 0 *to* system 1. $[C_{10}] = [C_{01}]'$, the transpose of $[C_{01}]$. $[I]_1$ is the inertia tensor of the body computed in system 1. $\mathbf{r}_1$ is the position vector of the center of mass of the body measured from 0_1, and $\mathbf{a}$ is the acceleration of 0_1 relative to $X_0Y_0Z_0$.

$\mathbf{M}$ is the sum of the moments, about 0_1, of all the external forces acting on the body. Equation (1) gives its components in system 1.

The curved brackets denote the operation of forming (either manually or by computer program), a column matrix

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